12.3 The Chain and General Power Rules

7) The Chain Rule

If $h(x) = g[f(x)]$, then:

$$h'(x) = \frac{d}{dx} g\big(f(x)\big) = g'(f(x)) \cdot f'(x)$$

Equivalently, if we let $U = f(x)$ we would have $h(x) = g(U)$ and thus $y = h(x) = g(U)$, then:

$$\frac{dy}{dx} = \frac{dy}{dU} \cdot \frac{dU}{dx}$$

Many composite functions have the form $h(x) = [f(x)]^n$ where n is a real number. In these cases a shortcut formula called the general power rule can be used to find $h'(x)$.

The General Power Rule

If the function f is differentiable and $h(x) = [f(x)]^n$ (n is a real number), then:

$$h'(x) = \frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} \cdot f'(x)$$

Example 1

Find $f'(x)$ for $f(x) = (x^2 - 4x + 3)^5$ using (a) the definition of the chain rule and (b) the general power rule.

Example 2

Find the derivatives of the following functions:

a) $f(x) = \sqrt[3]{2x^6 - 5x^2}$

b) $g(x) = 2x^2(7x^3 - 1)^8$

c) $h(x) = \left(\frac{x-5}{2x+1}\right)^{13}$

d) $f(x) = (6x-1)^4(2x^2+8x)^5$

e) $g(x) = \frac{(x^5 - x)^9}{(3x + 2)^7}$

f) $h(x) = \frac{4}{(7x^2 - 3x)^{10}}$